New defn    $A(0)\Omega$ is a localized state if

$A(0)\Omega \in$ range $P_X$ where $P_X \in R(O)$

$X$ is $\infty$-dim. subspace of $H$ corresponding to what can be measured locally.

But $P_X = 1 \in R(O)$

so this lets in all states, including $\Omega$.

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$A(0)\Omega$ not measurable.

What is measurable locally is $P_0 \in R(O)$

Compute $\mathrm{Prob}^{\Omega}(P_0 = 1)$

$= (\Omega, P_0\Omega) =$ ~~Norm (+~~

$\| P_0 \Omega \|^2$.

---

note all $P_{A(0)\Omega}$ are not $\in R(O)$

note $\forall A(0)$, $P_{A(0)\Omega} \notin R(O)$

what can measured locally corresponds to $P_0 \in R(O)$

N. W. Stiles

$$2/_3\,(x) = \int \frac{4^3 L}{\sqrt{u(\theta)}}\, e^{i\theta \cdot x}$$

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Queries re Malament's presentation

Garden - Variety

I will do 3 things

1. What does M's theorems mean
2. Give an alternative simple proof
3. Disagree with the conclusion

# Localization and the Vacuum

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## 1. Introduction

In a recent paper Malament ([1992]) has proved some very elegant theorems concerning the detection of particles in the vacuum state of a relativistic quantum field theory. Firstly he shows that there is a non-vanishing probability that a particle detector of a most general sort will 'fire', in response to its coupling to an initial vacuum state of the field. In a second theorem he shows the existence of correlations between the 'firing' of a localized detector and any other local observable in the field, irrespective of the separation of the two localizations in question. The object of the present paper is to investigate the significance of these theorems in the general context of understanding and interpreting relativistic quantum field theory.

## 2. The vacuum of a Relativistic Quantum Field

In a sense Malament is drawing attention to well-known, if somewhat paradoxical features of relativistic quantum field theory (RQFT). Malament obtains his result by using the famous Reeh-Schlieder theorem[1] in the form that the vacuum is cyclic for the whole Hilbert space of the field, with

× Question: to the intuitive understanding of the Reeh–Schlieder theorem.

respect to any local algebra associated with an arbitrary bounded open set in ~~space-time~~ Minkowski spacetime.

The intuitive idea here is that ~~any~~ ~~local observable~~ acting on the vacuum with ~~any~~ all the members of the local algebra R(O) attached to the bounded open set O will get us as close as we like to any state of the field.

This seems amazing since it seems to ~~say~~ say that performing operations in some ~~tiny~~ region O could generate excitations in the field that were localized in some ~~tiny~~ region O' that did not overlap with O, and was space-like separated from O by an arbitrarily large interval. But how could this happen in a local field theory? The answer is of course that the vacuum is a highly non-local state of the field. Intuitively, tweaking the vacuum over here can ~~to~~ produce ~~an~~ a response over there, not by action-at-a-distance, but 'by exploiting the correlations built' into ~~the~~ relativistic vacuum between distant events. Essentially Malament's second theorem is the vital clue to. ~~what is going on in his first theorem.~~

However it is important to realise that these vacuum correlations are not independent of distance, as

in Bell-type correlations but fall off exponentially with distance on a scale set by the Compton wave-length of a massive field, or the ordinary wavelength of a photon field.[2] It is well-known that the correlations maximally violate the Bell inequality, i.e. achieving the so-called Cirelson bound of $2\sqrt{2}$, against the classical limit of 2 for the Bell inequalities.[3]

Cirelson

Malament is quite right to say that the correlations are remarkably for any distance separating the localizations, but the exponential form of the distance dependence is of vital importance in assessing whether ~~using~~ vacuum correlations could be used[4] to perform a loophole-free version of the Bell experiment.

new para

But let us turn now directly to the first theorem. Malament presents this as an exercise in measurement theory. I shall give a different sort of proof of essentially the same result that does not discuss ~~the properties~~ of detectors at all.

new para

My own theorem is as follows:

X INSERTION A

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~~The action of any~~ element of any local algebra on the vacuum state can never produce a state orthogonal to the vacuum state.

Proof: Let $A(0)$ be any element of $R(O)$ for any $O$. We require to show that $(\Omega, A(O)\Omega) \neq 0$, where $\Omega$ is the vacuum state.

Denote $A(O)\Omega$ by $\chi$ and the projection operator onto $\chi$ by $P_\chi$

Assume $(\Omega, \chi) = 0$

$$\Rightarrow \quad |(\Omega, \chi)|^2 = 0$$

$$\Leftrightarrow \quad (\Omega, P_\chi \Omega) = 0$$

$$\Rightarrow \quad (\Omega, P_\chi^2 \Omega) = 0 \qquad \text{since } P_\chi = P_\chi^2$$

$$\Rightarrow \quad (P_\chi \Omega, P_\chi \Omega) = 0 \qquad \text{since } P_\chi \text{ is self-adjoint}$$

$$\Rightarrow \quad \| P_\chi \Omega \|^2 = 0$$

$$\Rightarrow \quad P_\chi \Omega = 0$$

But it follows as an easy consequence of the Reeh-Schlieder theorem that $\Omega$ is a separating vector for any local algebra associated with a bounded open set $O$.[4]

Hence we conclude from $P_\chi \Omega = 0$ that $P_\chi = 0$

But this is impossible since it would imply $(\chi, P_\chi \chi) = 0$ whereas the value of this expectation value is clearly one.

So by contradiction the theorem is proved.

## 3. The significance of the result

Klein-Gordon

Another way of stating our theorem is that the vacuum expectation value of any local observable is non-vanishing. One might wonder why the scalar field in Klein-Gordon theory which has the right micro-causality properties to be a candidate for an observable is not a counterexample. Clearly $(\Omega, \Phi(x)\Omega) = 0$, since we can write $\Phi(x) = \Phi^{(+)}(x) + \Phi^{(-)}(x)$ where $\Phi^{(+)}$ annihilates the vacuum and $\Phi^{(-)}$ creates a one-particle state orthogonal to the vacuum.

The answer, of course, is that $\Phi(x)$ is not a local observable in the sense of algebraic quantum field theory. Well, first the theory does not contemplate operators defined at a point but only so-called smeared fields, or more correctly distributions ~~that~~ but that is not the vital point. The vital point is that ~~local~~ observables in a local von Neumann algebra correspond to bounded operators whereas the fields, even smeared fields, are represented by unbounded operators. Of course one can always turn an unbounded operator into a bounded one by simply truncating its spectral expansion, but if one does then the vanishing of its vacuum expectation value is

no longer true: so our theorem is not violated by any bounded ~~observable~~ local observable.

new para

A third way of stating our theorem which relates it directly to Malament's first theorem is to notice that $A(0)\Omega$ ~~can approximate as near as we like any state $\chi$ of the field~~ is a state of the field, call it $\chi_{A(0)}$ that can approximate as near as we like any state of the field. So our theorem says

Display

$$\mathrm{Prob}(\Omega \to \chi_{A(0)}) = |(\chi_{A(0)}, \Omega)|^2 \neq 0$$

or in other words there is a non-vanishing probability of finding the state $\chi_{A(0)}$ if we are in the vacuum state $\Omega$ where $\chi_{A(0)}$ is as close as we like to any state of the field.

new para

Notice that our theorem does not say $\mathrm{Prob}(\Omega \to \chi) \neq 0$ for any state $\chi$. This is clearly not true for the one-particle states, two-particle states etc. which of course are all orthogonal to $\Omega$. What our theorem does say is that local operators can never produce pure many-particle states from the

~~vacuum. There is always left behind a trail of the vacuum state which is what keeps Prob ($\Omega \to \chi_{A(0)}$) from being over exactly zero.~~

new para

~~Another way of putting this is that pure many-particle states in RQFT are themselves spatially non-local entities which cannot be produced by 'local operations' even on a non-local state ~~such as the vacuum itself~~.~~

Malament correctly derives the resolution of a counter-example to his theorem provided by the Unruh effect, in the circumstance this his result, like mine, only applies to local operations, pure particle detectors have to be coupled to the field over unbounded regions of space but.

So local detectors are not really detecting particles — they are firing alright, but in response to localized aspects of the field, not their global aspects, which are connected to what may properly called particle states.

2e new para

There are a couple other points of worth to make:

(1) Where does the energy come from to fire the detector? Essentially the detector has to feed energy into the field, exactly as happens with

the detectors in the Unruh effect.
It is a bit ~~measuring the kinetic energy of an electron~~ like detecting an ~~particle~~ electron in a region outside the nucleus of an atom where it ought to have negative kinetic energy. The ~~energy~~ observation feeds energy into the system so that any subsequent measurement of the kinetic energy would always be positive!

(2) We cannot exploit the vacuum correlations to convey information because they are just like the Bell correlations. As shown very clearly by Licht (1966), selective operations in $\mathcal{O}$ are required to produce arbitrary operations in $\mathcal{O}'$ — just looking on non-selective measurements devices won't do the job.

## 4. Conclusion

Malament has given very elegant proofs of two theorems which highlight some features of the vacuum ~~of a ...~~ in RQFT, that could be thought paradoxical, but there are not really any paradoxes, just some remarkable physics.

Footnotes

1. See Reeh and Schlieder (1961).

2. See Fredenhagen (1985)
and Summers and Werner (1985)

3. See Landau (1987).

4. See, for example, Streater and Wightman (1989).
~~p. 139~~. Theorem 4.3, p. 139.

# References


7 ~~6~~ WERNER, R. and SUMMERS, S.J. (1985): 'The Vacuum Violates ~~the~~ Bell's Inequalities', Physics Letters A 110, 257–259

(3) LICHT, A.L. (1966): 'Local States', Journal of Mathematical Physics 7, 1656–1669.

(4) MALAMENT, D. (1992): 'Finding Particles in the Vacuum state of a Quantum Field', forthcoming.

(2) LANDAU, L. (1987): 'On the Non-Classical Structure of the Vacuum', Physics Letters A 123, 115–118.

(5) REEH, H. and SCHLIEDER, S. (1961): 'Bemerkungen zur Unitäräquivalenz von Lorentzinvarianten Feldern', Nuovo Cimento 22, 1051–1068.

(1) ~~K.~~ FREDENHAGEN, K. (1985): 'A Remark on the Cluster Theorem', Communications in Mathematical ~~461–463~~. Physics 97, 461–463.

6 ~~5~~ STREATER, R. AND WIGHTMAN, A. (1989): 'PCT, Spin and Statistics, and All That', Addison-Wesley, New York.